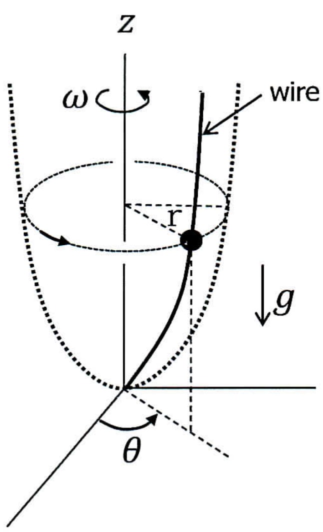
소속대학원		수험번호		성 명		감독교수 확 인	(인)

2012학년도 석사과정/석사박사통합과정 후기모집 면접·구술고사 전공시험

과목명 : 고전역학/전자기학 2012. 05. 25 시행

1. (20 points) Let us consider a bead (구슬) of mass m which slides freely without any friction along a smooth wire bent in the shape of a curve $z = cr^n$ with $n \geq 1$ and $c > 0$ in the figure. Here the wire is assumed to be rotating about its vertical axis at a constant angular velocity ω. A constant gravitational force is applied downward to the bead.

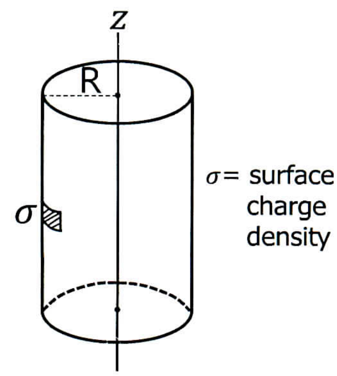


(a) Write down the Lagrangian of this system.

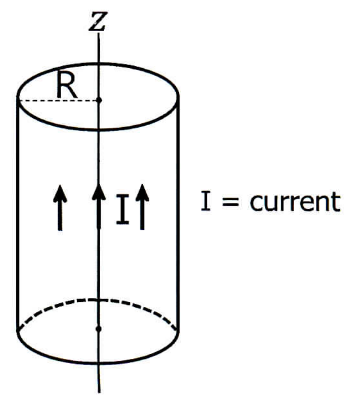
(b) Obtain the equation of motion for r.

(c) Assuming that the bead is rotating with constant radius $r = R$, obtain the angular velocity ω in terms of R, m, g, c, n. Find a value of n which makes ω independent of R and m.

2. (30 points) Let us consider a cylindrical wire of permeability μ. Assume that this wire is infinitely long. The radius of the wire is R.



(a) Assume that this wire is a very good conductor. The charge density on the surface is σ. Obtain the electric field vector $\vec{E}$ as a function of radius r from the inside ($r < R$) to the outside ($R < r$) of wire.



(b) Now let us remove all the surface charges of the wire and then let it carry a steady and uniform current I. Obtain the magnetic field vector $\vec{B}$ (magnetic flux density) as a function of radius from the inside to the outside of the wire.

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3. (20 points) Consider a system in which atoms are located in a regular lattice and each atom has a spin 1/2 and associated intrinsic magnetic moment $\mu_0>0$. Assume that each atom interacts only weakly with other atoms so that all the other atoms act as a heat reservior. If the system is placed under an external magnetic field H_0 along the z-direction then the Hamiltonian is given by

$$H=-\mu_0 H_0 \sum_i \sigma_i^z$$

where σ_i^z is the z-component of the Pauli matrix at the site i. For $\mu=\pm\mu_0$, the corresponding magnetic energy of the atom is $\epsilon=\mp\mu_0 H_0$.

(a) Calculate the mean magnetic moment $\overline{\mu}$.

(b) Draw $\overline{\mu}/\mu_0$ as a function of $\mu_0 H_0/k_B T$

where k_B is the Boltzmann constant. Discuss $\mu_0 H_0/k_B T \gg 1$ and $\mu_0 H_0/k_B T \ll 1$ limits.

(c) Calculate the susceptibility per atom

$\chi=\dfrac{\partial\overline{\mu}}{\partial H_0}$. Discuss $\mu_0 H_0/k_B T \gg 1$ and $\mu_0 H_0/k_B T \ll 1$ limits.

※ If necessary, use the following formula:

$$\sinh(x)=x+\frac{x^3}{3!}+\cdots,\ \cosh(x)=1+\frac{x^2}{2!}+\cdots,$$

$$\tanh(x)=x-\frac{x^3}{3}+\cdots,\ \coth(x)=\frac{1}{x}+\frac{x}{3}+\cdots \text{ for } |x|\ll 1.$$

4. (30 points) Consider a two-level system with the following Hamiltonian:

$$H=\begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}$$

where λ is a non-zero positive number.

(a) Find the energy eigenvalues and eigenstates.

(b) If the system starts in $\Psi(0)=\begin{pmatrix}1\\0\end{pmatrix}$ state at $t=0$, what is its state $\Psi(t)$ at time $t>0$?

(c) Obtain the probability to find $\begin{pmatrix}0\\1\end{pmatrix}$ state at a time $t>0$. What is the period of the probability oscillation?