소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 양자역학 **2022 . 07. 29 시행**

1. [50 pts] Consider monoenergetic beam of spin 1/2 particles of mass m and energy E moving in x direction. The potential energy operator is given by,

$$V(x) = V_0 - \gamma B_0 S_z \text{ for } x > 0$$

$$V(x) = 0 \text{ for } x \le 0$$

where $V_0 > 0$ is a positive constant potential, γ is gyromagnetic ratio, $B_0 > 0$ is magnitude of external magnetic field applied in z direction, and S_z is z direction spin angular momentum operator.

(a) [7 pts] Write the Hamiltonian for the particles in the x > 0 region and sketch the potential energy as a function of x for particles having z direction spins up and down.

(b) [7 pts] Suppose that spin of particles coming from $-\infty$ are eigenstate of x direction spin angular momentum operator S_x with eigenvalue $+\hbar/2$ and have energy $E > V_0 > 0$. Write down the general eigenstate of such an incoming beam considering orbital and spinor part.

(c) [10 pts] Write down the general solution of transmitted and reflected beams ($E > V_0 > 0$).

(d) [10 pts] What are the boundary conditions at x = 0 ? Using the boundary conditions, write down the equations that must be satisfied by the amplitudes appearing in part (b) and (c).

(e) [8 pts] If $E = V_0 > 0$, what is the probability of measuring z direction spin angular momentum $+\hbar/2$ for the transmitted beam ?

(f) [8 pts] Instead of incoming beam with x direction spin polarization, if we start with unpolarized incoming beam (but still $E = V_0 > 0$), what is the probability of measuring z direction spin angular momentum $+\hbar/2$ for the transmitted beam ?

2. [50 pts] Consider a two-level system. An operator $\hat{A}$ representing the observable A has normalized eigenstates ψ_1 and ψ_{-1} with eigenvalues +1 and -1, respectively. Another operator $\hat{B}$ for the observable B has eigenstates $(\psi_1 + \sqrt{3}\psi_{-1})/\sqrt{2}$ and $(\sqrt{3}\psi_1 - \psi_{-1})/\sqrt{2}$ with eigenvalues +1 and -1.

(a) [12 pts] Calculate $[\hat{A}, \hat{B}]$. Express it as a 2×2 matrix in the basis $\{\psi_1, \psi_{-1}\}$.

(b) [12 pts] Suppose that a system is in the state $(\psi_1 + \psi_{-1})/\sqrt{2}$. . Calculate the variances $(\Delta\hat{A})^2 = <\widehat{A^2}> - <\hat{A}>^2$ and $(\Delta\hat{B})^2$. Do they satisfy the uncertainty relation?

(c) [12 pts] Suppose the observable A is measured at some time. Immediately after this, the expectation value of B is measured to be 1/2. What was the measured value of A?

(d) [14 pts] For some normalized state $\overline{\psi} \equiv a\psi_1 + b\psi_{-1}$ with a and b real and $|a|, |b| < 1$, find all the values of a that minimize $(\Delta\hat{A})^2$. Explain why those values do so.

자격시험 문제

과목명 : 통계역학 2022 . 07. 29 시행

1. (50pts) Consider non-interacting photon gas in a three-dimensional container. Answer the following questions.

[Hint: The first law of thermodynamics is $dU = TdS - PdV$, where U is the average energy, T is the temperature, S is the entropy, P is the pressure, and V is the volume.*]*

(a) Show that the entropy S and the pressure P can be obtained from the Helmholtz free energy A by the relationships, $S = -\left(\frac{\partial A}{\partial T}\right)_V$ (5pts) and $P = -\left(\frac{\partial A}{\partial V}\right)_T$ (5pts).

(b) The Helmholtz free energy of the photon gas is given by $A = -\frac{\pi^2 V (k_B T)^4}{45\hbar^3 c^3}$ in three dimensions. Find the entropy S (5pts) and the pressure P (5pts) of the photon gas, using the relationship you obtained in (a).

(c) Find the heat capacity at constant volume, C_V (15pts).

(d) Show that in the adiabatic process the photon gas exhibits the pressure-volume relation of $PV^{\gamma} = (constant)$, and find the value of γ (15pts).

2. (50pts) Consider an one-dimensional harmonic oscillator potential $V(x) = m\omega^2 x^2/2$ in which N identical fermions of mass m are trapped. Suppose the fermions do not interact and their spins are polarized to the same values. The system is at thermal equilibrium of temperature T.

(a) At $T = 0$, the system is in its ground state. What is the energy of the highest occupied state of a particle (5pts)? What is the total energy E_G of the ground state (5pts)?

(b) The excited states of the system have energies $E_m = E_G + m\hbar\omega$, where m are integers. For each m, there are $\Omega(m)$ number of states having energy E_m. Count $\Omega(m)$ for $m = 1, 2,$ and 4 (5pts each).

(c) For large $m \gg 1$, the number of states $\Omega(m)$ can be approximated to

$$\Omega(m) \approx \frac{e^{\pi\sqrt{2m/3}}}{4\sqrt{3}m}.$$

With this approximation, compute the entropy of system $S(E)$ as a function of energy E. (5pts) Also, show that the ground state, $E - E_G$, is proportional to T^2 and obtain the proportionality factor (5pts).

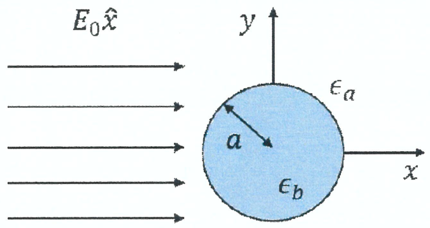
(d) By using the result of (c) above, obtain the entropy as a function of T (5pts), and compute the heat capacity C_V (5pts). Identify that the energy increase from the ground state, $E - E_G$, is proportional to T^2, and obtain the proportionality factor (5pts).

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 전기역학 **2022 . 07. 29 시행**

1. [40 pts] A uniform electric field $E_0\hat{x}$ exists in a dielectric having permittivity ϵ_a. With its axis perpendicular to this field, a sufficiently long cylindrical dielectric rod (extended along the z-direction) having permittivity ϵ_b and radius a is introduced.



Using a cylindrical coordinate (ρ,θ,z), we will look for solutions of Laplace equation inside $(\phi_b(\rho,\theta))$ and outside $(\phi_a(\rho,\theta))$ of the rod with the following boundary conditions.

(i) $\phi_a = \phi_b$ at $\rho = a$

(ii) $\epsilon_a \dfrac{\partial\phi_a}{\partial\rho} = \epsilon_b \dfrac{\partial\phi_b}{\partial\rho}$ at $\rho = a$

(iii) $\phi_a \to -E_0 x = -E_0\rho\cos\theta$ for $\rho \gg a$

(a) [10 pts] Justify the boundary conditions (ii) and (iii).

(b) [20 pts] The general forms of the potential outside and inside the rod satisfying the boundary conditions are

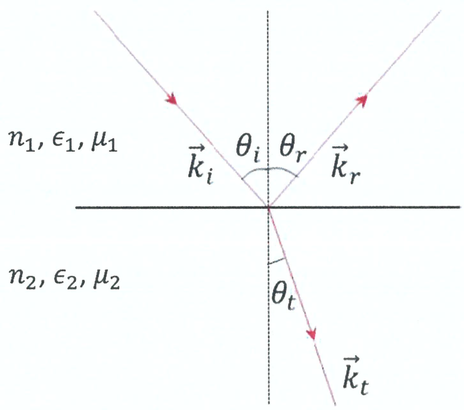
$$\phi_a = -E_0\rho\cos\theta + \sum_{k=1}^{\infty}\rho^{-k}(C_k\cos k\theta + D_k\sin k\theta)$$

$$\phi_b = \sum_{k=1}^{\infty}\rho^{k}(A_k\cos k\theta + B_k\sin k\theta)$$

Determine the potential and the electric field at points outside and inside the rod, neglecting the end effects (Set $\phi = 0$ on y-z plane).

(c) [10 pts] Sketch the electric field lines inside and outside the rod when $\epsilon_a > \epsilon_b$.

2. [60 pts] Consider two nonconducting media $(\sigma = 0)$ described by the refractive index n, permittivity ϵ, and permeability μ for the medium 1 $(n_1,\ \epsilon_1,\ \mu_1)$ and medium 2 $(n_2,\ \epsilon_2,\ \mu_2)$. A plane wave in medium 1 is incident on medium 2 at an angle θ_i. The incident, reflected, and transmitted electric fields are given by $\vec{E}_i = \vec{E}_{0i} e^{i(\vec{k}_i\cdot\vec{x} - \omega_i t)}$, $\vec{E}_r = \vec{E}_{0r} e^{i(\vec{k}_r\cdot\vec{x} - \omega_r t)}$ and $\vec{E}_t = \vec{E}_{0t} e^{i(\vec{k}_t\cdot\vec{x} - \omega_t t)}$, respectively.



(a) [20 pts] For the given incident angle θ_i, find the reflected angle θ_r and the transmitted angle θ_t.

(b) [20 pts] Find the condition that the incident wave is totally reflected.

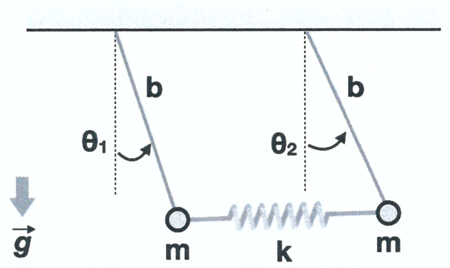
(c) [20 pts] Find the condition that the incident wave is totally transmitted. Consider two cases that the electric field is 1) parallel and 2) perpendicular to the plane of incidence, respectively.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확　　인	(인)

자격시험 문제

과목명 : 고전역학 　　　　　　　　　　　　**2022. 07. 29 시행**

1. [30 pts] Two pendula are connected by a massless spring of force constant k. Each pendula is made up of a massless rod of length b and a mass m attached to the rod. The two pendula swing in the same plane vertical to the ground in the uniform gravitational field g. The spring is unstretched in the equilibrium position.



(a) [10 pts] Determine the Lagrangian of the system using the generalized coordinates θ_1 and θ_2 shown in the figure.

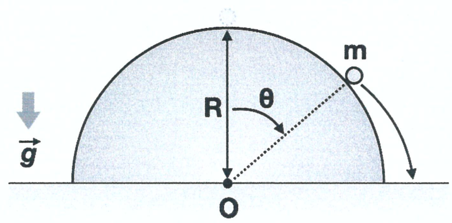
(b) [10 pts] Find Lagrange's equations of motion of the system about the equilibrium points $\theta_1 = \theta_2 = 0$ assuming small oscillations (θ_1, $\theta_2 \ll 1$).

(c) [10 pts] Using your answer in (b), find the eigenfrequencies and describe the normal mode motion. You will need to write the two angular displacements as functions of time, i.e., $\theta_1(t)$ and $\theta_2(t)$.

2. [70 pts] In this problem, two different objects start from rest at the top of a fixed hemisphere of radius R, with a negligible initial speed.

First, consider a sizeless particle of mass m sliding down the *frictionless* surface of the hemisphere. See Figure (i) below.

(i)



(a) [10 pts] To consider the possibility of the particle leaving the hemisphere's surface, we use the polar coordinates (r, θ) for its position, where r is the distance between the particle's center and point O. Determine the Lagrangian of the particle.

(b) [10 pts] With the constraint equation, $f(r,\theta) = r - R = 0$, find two Lagrange's equations with a multiplier λ (i.e., $\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \lambda \frac{\partial f}{\partial q_j} = 0$ where q_j is each of the generalized coordinates).

(c) [10 pts] Applying the constraint $r = R$ to your answer in (b), express λ as a function of θ. (Hint: you will need to prove $\dot{\theta}^2 = \frac{2g}{R}(1 - \cos\theta)$ from energy conservation or by direct integration.)

(d) [5 pts] By identifying the physical meaning of λ (*"constraint force"*), find $\cos\theta_c$ where θ_c is the angle at which the particle leaves the surface.

[Problem 2 continues in the next page.]

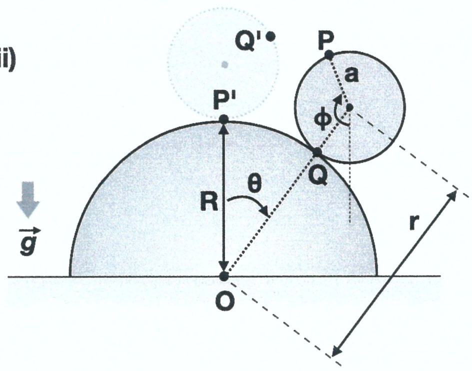
소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 고전역학 2022. 07. 29 시행

Now, consider a uniform, solid sphere of mass m and radius a that begins to *roll without slipping* from the top of the hemisphere. See Figure (ii).

(ii)



(e) [10 pts] Again, r is the distance between the sphere's center and point O, while ϕ is the rotational angle of the sphere with respect to the vertical. Determine the Lagrangian of the sphere. The rotational inertia of the sphere is $I=\frac{2}{5}ma^2$.

(f) [10 pts] With the two constraint equations for the described motion, $f_1(r,\theta,\phi)=r-(R+a)=0$ and $f_2(r,\theta,\phi)=a(\phi-\theta)-R\theta=0$, find three Lagrange's equations with multipliers λ_1 and λ_2 (i.e., $\frac{\partial L}{\partial q_j}-\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j}+\sum_k \lambda_k \frac{\partial f_k}{\partial q_j}=0$).

(g) [10 pts] Applying the constraints $r=R+a$ and $a(\phi-\theta)=R\theta$ to your answer in (f), express λ_1 as a function of θ. Note that λ_1 corresponds to the first constraint equation, $f_1(r,\theta,\phi)=r-(R+a)=0$.

(h) [5 pts] By identifying the physical meaning of λ_1 ("*constraint force*"), find $\cos\theta_c$ where θ_c is the angle at which the sphere leaves the surface.