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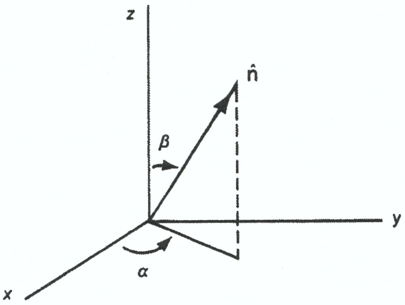
자격시험 문제

과목명 : 양자역학 **2021. 07. 30 시행**

1. [40pt] Consider a two-state Hamiltonian $H = \omega \hat{n} \cdot \vec{S}$ with the direction $\hat{n}$ defined in the figure and $\vec{S}$ being the spin operator.



[a] [10pt] Show that the state $|\chi> = \begin{pmatrix} \cos\frac{\beta}{2} \\ \sin\frac{\beta}{2} e^{i\alpha} \end{pmatrix}$ is an energy eigenstate. What is its energy eigenvalue?

[b] [10pt] Derive the above eigenstate using rotations of a spin-1/2 system. [Hint: Rotate the spin-up state.]

[c] [5pt] Suppose $\hat{n} = \hat{z}$. The initial state at $t = 0$ is $|a(0)> = \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$ with $a_\pm$ being real. Find $|a(t)>$ at time $t > 0$.

[d] [5pt] For $|a(t)>$, calculate the expectation value of the spin measurement along the $\hat{z}$-axis. Calculate using both Schrodinger and Heisenberg pictures.

[e] [10pt] Calculate the expectation values of the spin measurements along $\hat{x}$ and $\hat{y}$-axes too, in the Heisenberg picture.

2. [40pt] Consider a particle with charge e and mass m, confined to move in a circle of radius ρ_0 on the $\rho - \phi$ plane. [Cylindrical coordinate $\vec{r} = (\rho, \phi, z)$.]

[a] [6pt] Write down the one-dimensional Hamiltonian along the circle, and solve it for the energy eigenfunction $\psi_0(r)$ and the eigenvalue E_0. Derive quantization conditions, if any. [No magnetic field yet.]

[b] [4pt] Now an external magnetic field $\vec{B} = B\hat{z}$ is turned on along the z-axis within the cylindrical region of radius ρ_c (with $\rho_c < \rho_0$). In the presence of electromagentic fields, path integral dictates an extra phase to the wavefunction $\psi_0(r)$

$$\frac{ie}{\hbar c}\int_0^r d\vec{r} \cdot \vec{A}.$$

For $\vec{A} = \nabla \Lambda(r)$ with some scalar function $\Lambda(\vec{0}) = 0$ at the reference point $\vec{0}$, express this phase in terms of $\Lambda(r)$. [Ignore scalar potentials.]

[c] [7pt] Write down the Hamiltonian. Show that the phase-supplied wavefunction still satisfies the Schrodinger equation.

[d] [8pt] The gauge is chosen in the following form

$$\vec{A} = \begin{cases} \dfrac{A_{\phi,out}}{\rho}\hat{\phi} & \text{for} \quad \rho > \rho_0 \\ \rho A_{\phi,en}\hat{\phi} & \text{for} \quad \rho \leqq \rho_0 \end{cases}$$

with constant $A_{\phi,out}$ and $A_{\phi,en}$. First, why does this form make sense? Second, find $A_{\phi,en}$ in terms of B, and match $A_{\phi,out}$ to $A_{\phi,en}$. Third, find $\Lambda(r)$ at $\rho > \rho_0$. [Use: $\nabla \times \vec{V} = -\dfrac{\partial V_\phi}{\partial z}\hat{\rho} + \dfrac{1}{\rho}\dfrac{\partial(\rho V_\phi)}{\partial \rho}\hat{z}$.]

[e] [10pt] For a close-loop along the circle, what condition must the phase satisfy? What does this mean to the magentic flux through the loop?

[f] [5pt] The phenomenon considered in this problem is known to be purely quantum mechanical. What aspects cannot be described by classical mechanics? Then, how can they be described by quantum mechanics? [Discuss qualitatively.]

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자격시험 문제

과목명 : 통계역학 **2021. 07. 30 시행**

1. [50 pts] The $N+1$ containers M_0, M_1, ..., M_N are filled with water. The temperature of M_n is given by T_n, with $T_n/T_{n-1}=\alpha>1$ for $n=1,\ 2,\ \ldots,\ N$. In the initial state, Object A with constant heat capacity C is in thermal equilibrium with M_0. We first move A to M_1 and wait until they reach thermal equilibrium. Then, in a similar manner, we move A from one container to the next in the order of M_1, M_2, ..., M_N. Throughout this "forward" process, there is no heat exchange between the system (A and the containers) and the surroundings. Besides, the amount of water in each container is so large that its temperature effectively remains constant.

(a) [15 pts] Calculate the entropy change of the system between the initial and the final states of the forward process described above.

(b) [15 pts] Now suppose that we perform the above process backward, moving A from M_N through M_{N-1}, ..., M_1 to M_0. What is the entropy change of the system through this "backward" process?

(c) [10 pts] Consider a composite process which consists of the forward process followed by the backward process. What is the sign of the total entropy change over this composite process? Is the composite process reversible?

(d) [10 pts] Now let us take the limit $N\to\infty$, while keeping T_0 and T_N constant. How does this modify the total entropy change discussed in (c)? Is the composite process reversible?

2. [50 pts] Planck assumed that, at a given frequency ν, the electromagnetic waves within a cavity absorb and emit energy only by discrete energy quanta, each having energy $\varepsilon=h\nu$. Thereby he derived the spectral density of black-body radiation, which we reproduce here.

(a) [10 pts] Find the total number Ω of the ways of distributing M indistinguishable energy quanta among N distinguishable modes.
(Hint: Putting five stars into four bins can be visualized as inserting two bars in between a series of five stars, *e.g.*, *||****| meaning 1, 0, 4, and 0 stars in the four bins.)

(b) [10 pts] Assume that the numbers M and N are large enough. Denoting by $u=\varepsilon M/N$ the mean energy of each mode, show that the entropy per mode is given by

$$s=k_{\mathrm{B}}\left[\left(1+\frac{u}{\varepsilon}\right)\ln\left(1+\frac{u}{\varepsilon}\right)-\frac{u}{\varepsilon}\ln\frac{u}{\varepsilon}\right].$$

(Hint: You may use Stirling's approximation $\ln N!\approx N\ln N-N$ for $N\gg 1$.)

(c) [10 pts] Express u as a function of the temperature T and the energy quantum ε.

(d) [10 pts] If the cavity has a cubic shape with the side length given by L, the allowed frequencies of the electromagnetic waves are given by

$$\nu=\frac{c}{2L}\sqrt{n_x^2+n_y^2+n_z^2},$$

where n_x, n_y, and n_z are positive integers. Using this relation, show that the number of modes in the frequency interval $[\nu,\ \nu+d\nu]$ can be expressed as $8\pi L^3\nu^2 d\nu/c^3$.

(e) [10 pts] Using the above results, show that the mean energy density of the black-body radiation in the frequency interval $[\nu,\ \nu+d\nu]$ is given by

$$u_\nu=\frac{8\pi h\nu^3}{c^3}\frac{1}{\mathrm{e}^{\beta h\nu}-1}.$$

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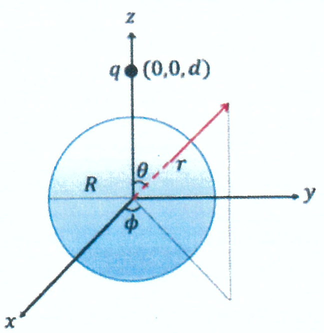
자격시험 문제

과목명 : 전기역학 2021. 07. 30 시행

1. [50 pts] The main idea of method of images is to add fictitious charges so that together with the actual charges they satisfy the boundary condition of the original problem. Consider a point charge q at $(0,0,d)$ outside a grounded conducting sphere of radius R at the center $(d > R)$.



(a) [10 pts] Suppose we add a single image charge q' at a distance d' from the center. Find q' and its location.

(b) [10 pts] Find the potential at an arbitrary position outside of the sphere.

(c) [10 pts] Draw the electric field schematically. Assume $q > 0$.

(d) [10 pts] Find the surface charge density of the sphere at an arbitrary point.

(e) [10 pts] What is the total induced charge on the surface of the sphere?

2. [50 pts] We want to study physics of an electron confined in a two-dimensional plane in magnetic field.

(a) [10 pts] Consider a circular ring of radius R, carrying a current I counterclockwise. The ring lies on xy-plane and is centered at (0,0,0). Draw qualitatively the magnetic field lines in the (x,0,z) plane and (x,y,0) plane.

(b) [10 pts] By integrating the Biot-Savart's law, find the z-component of the magnetic field in the symmetric axis.

(c) [10 pts] Consider next a solenoid of radius R, infinite height, and N turns per meter along z-direction, carrying a current I. Draw qualitatively the magnetic field lines. Calculate the magnetic field B around the solenoid.

(d) [5 pts] Draw qualitatively the magnetic field lines for a real solenoid with a finite height.

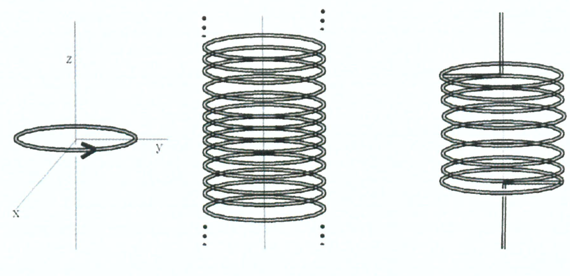
(e) [15 pts] Consider an electron moving in the xy-plane at z=0. The infinite solenoid is centered at (0,0,0). The motion of the electron is confined by a mechanical potential $V(x,y) = \frac{1}{2}K(x^2 + y^2)$.

What is the equation of motion of the electron? Discuss trajectory of the electron for the following three initial conditions:

(i) $\vec{x_0} = (r_0 > R, 0),\ \vec{v_0} = (0,0)$

(ii) $\vec{x_0} = (r_0 < R, 0),\ \vec{v_0} = (0,0)$

(iii) $\vec{x_0} = (r_0, 0),\ \vec{v_0} = (0, v_0)$



(a, b) (c) (d)

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자격시험 문제

과목명 : 고전역학 **2021. 07. 30 시행**

[문제-1] (20점) 질량 m의 입자가 중심력 $\vec{F} = -c^2 \frac{\vec{r}}{r^{5/2}}$을 받고 운동하고 있다.

(가) 포텐셜 에너지를 계산하시오. (2점)

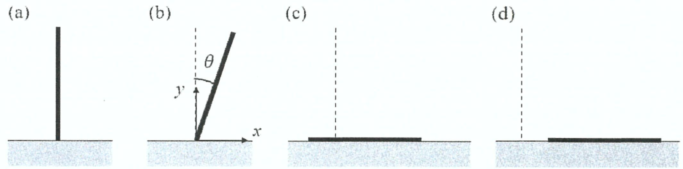
(나) 이 계의 라그랑지안을 spherical coordinate 으로 기술하고 운동방정식을 유도하시오. (4점)

(다) 이 입자가 원 궤도 운동을 하는 경우 궤도 반경을 각운동량으로 나타내고 이의 주기를 구하시오. (6점)

(라) Radial 운동방정식에 대한 유효 포텐셜의 그래프를 그리시오. (3점)

(마) (다)의 원 궤도에 약간의 섭동이 생겼을 경우, radial 방향으로 작은 진동이 일어난다. 이 진동에 대한 진동 주파수를 구하시오. (5점)

[문제-2] (30점) 아래 그림 (a)와 같이 연필이 수직으로 서 있다가 그림 (b)와 같이 쓰러질 때 매끈한 면에서는 그림 (c)와 같이, 거친 면에서는 그림 (d)와 같이 쓰러진다. 연필을 질량 m, 길이 L인 균질의 가느다란 막대로 간주하고 질점과 바닥면 사이의 최대정지마찰계수를 μ_s 라 하자. 중력가속도는 g이다.



(가) 연필이 그림 (b)와 같이 각 θ로 기울었을 때의 각속도와 각가속도를 θ의 함수로 구하시오. (6점)
(힌트: 막대의 한쪽 끝에 대한 관성모멘트는 $I = mL^2/3$ 이고, 질량중심의 위치는 $x = (L/2)\sin\theta$, $y = (L/2)\cos\theta$ 로 주어진다.)

(나) 그림 (b)의 상태에서 바닥면이 연필에 작용하는 힘의 x-성분 F_x 와 y-성분 F_y 를 각각 구하시오. (8점)

(다) $r(\theta) = \frac{F_x}{F_y}$ 의 그래프를 정성적으로 그리시오. (6점)

(라) μ_s 의 값에 따라 그림 (c)와 그림 (d)에서 일어나는 현상을 (다)의 그래프를 이용하여 설명하시오. (5점)

(마) 그림 (d)와 같이 쓰러지기 위한 μ_s 의 최소값을 구하시오. (5점)