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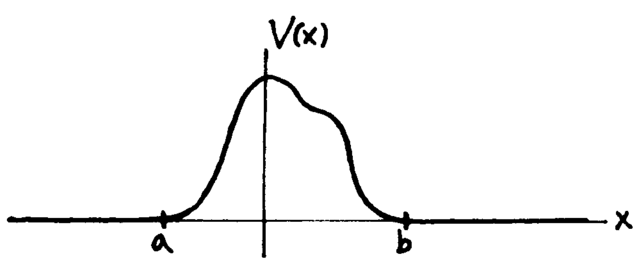
자격시험 문제

과목명 : 양자역학 **2019 . 07. 19 시행**

1. Consider one-dimensional time-independent potential $V(x)$, which is non-zero only for $a < x < b$. A particle with the mass m and the wavenumber $k = \sqrt{2mE}/\hbar$ has wavefunction

$$\psi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & x < a \\ \psi_{ab}(x) & a < x < b, \\ Ce^{ikx} + De^{-ikx} & x > b \end{cases}$$

where coefficients A, B, C, D are complex constants and some function $\psi_{ab}(x)$.



(a) [10pt] Show that the probability current

$$j(x,t) = \frac{\hbar}{2mi}[\psi^*(x,t)\frac{\partial \psi(x,t)}{\partial x} - \frac{\partial \psi^*}{\partial x}\psi]$$

must have the same value at all x. What does this physically mean?

(Hint: Consider $\frac{\partial j(x,t)}{\partial x}$.)

(b) [10pt] What condition do A, B, C, D must satisfy for this?

(c) [10pt] Let

$$\begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}\begin{pmatrix} A \\ D \end{pmatrix} \equiv S\begin{pmatrix} A \\ D \end{pmatrix}.$$

Show that the matrix S is unitary, i.e. $S^\dagger S = 1$.

(d) [10pt] The Hamiltonian including the arbitrary potential is time-reversal symmetric. Show that S is a symmetric matrix too, i.e. $S_{ij} = S_{ji}$.

(e) [10pt] Show that the transmission probability T is given by $T = |S_{12}|^2$ for both left-incoming and right-incoming wave.

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자격시험 문제

과목명 : 양자역학 **2019 . 07. 19 시행**

2. [50 pts] The Hamiltonian and energy eigenvalues of a one-dimensional (1D) quantum harmonic oscillator, of mass m and spring constant k, are given by

$$\hat{H}=\frac{\hat{p}^2}{2m}+\frac{1}{2}k\hat{x}^2,$$

$$E_n=(n+\frac{1}{2})\hbar\omega$$

$$(n=0,1,2,\dots \text{ and } \omega=\sqrt{k/m}).$$

(a) [5 pts] Suppose that a particle of mass m_1 is present under a harmonic potential of spring constant k_1, and another particle with mass m_2 is under a harmonic potential of k_2. Write down the Schrödinger equation describing the total system of the two particles. (Assume the particles are not interacting with each other.)

(b) [5 pts] What are the energy eigenvalues of the system.

(c) [10 pts] Suppose that $m_1=m_2=m$ and $k_1=k_2=k$. What is the degeneracy of the state whose energy eigenvalue is $E=(n+1)\hbar\omega$?

(d) [10 pts] The problem in (c) is equivalent to the problem to describe a single 2D quantum harmonic oscillator of mass m and spring constant k. Then, show that the angular momentum of the particle, $\hat{L}=\hat{x}_1\hat{p}_2-\hat{x}_2\hat{p}_1$, is the constant of motion.

(e) [20 pts] Show that, if the energy eigenvalue of a state is $E=(n+1)\hbar\omega$, the possible eigenvalues of the angular momentum $\hat{L}$ are $m_z\hbar$, where $m_z=n,n-2,n-4,\dots,-n$.

[Hint: Try to express $\hat{H}$ and $\hat{L}$ in terms of

$$\hat{a}_R=\frac{1}{\sqrt{2}}(\hat{a}_1+i\hat{a}_2) \text{ and}$$

$$\hat{a}_L=\frac{1}{\sqrt{2}}(\hat{a}_1-i\hat{a}_2)$$

, where

$$\hat{a}_i=\sqrt{\frac{m\omega}{2\hbar}}\hat{x}_i+\frac{i}{\sqrt{2\hbar m\omega}}\hat{p}_i \qquad (i=1,2).$$]

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자격시험 문제

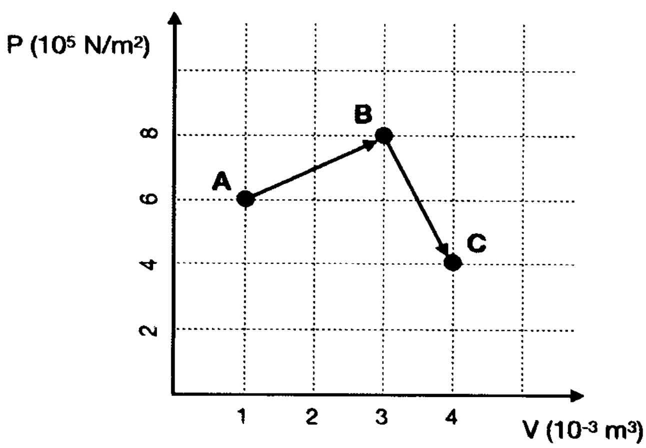
과목명 : 통계역학 2019. 07. 19 시행

1. [30 pts] A heat absorbed by a mole of ideal gas in a quasi-static process in which its absolute temperature T changes by dT and its volume V by dV is given by

$$\tilde{d}Q = C_V dT + \bar{p} dV$$

where C_V is the molar heat capacity at constant volume, and $\bar{p} = RT/V$ is its mean pressure with the gas constant $R=$ 8.314 Joule/(mol · K).

Now, an ideal diatomic gas has a molar internal energy of $E = \frac{5}{2}RT$. A mole of this gas is taken quasi-statically from state A to B, then to C, along the straight lines shown in the $p-V$ diagram below.



(a) [5 pts] What is C_V for this gas?

(b) [5 pts] What is the work (in Joule) done by the gas in the process from A to B, then to C?

(c) [10 pts] What is the heat (in Joule) absorbed by the gas in the process?

(d) [10 pts] Show that the change of entropy of the gas in the process can be expressed as

$$\Delta S = C_V \ln\left(\frac{T_C}{T_A}\right) + R \ln\left(\frac{V_C}{V_A}\right),$$

and evaluate its numerical value accurate to only two significant figures (in Joule/K). One may want to use one or more of the followings: ln(2)=0.7, ln(3)=1.1, ln(5)=1.6

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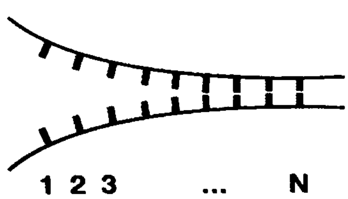
자격시험 문제

과목명 : 통계역학 **2019. 07. 19 시행**

2. [70 pts] The formation or unraveling of certain biomolecules (such as double-stranded DNA) can be described by an one-dimensional zipper model. Consider a zipper of N parallel segments (links), each of which can be closed with the ground-state energy 0, or open with energy ϵ (> 0). The zipper is immersed in a heat reservoir of absolute temperature T. The Boltzmann constant is k_B.

First, consider the case where the zipper may only unzip successively from the left end. To put it another way, the n-th segment can only be open if all the segments to its left (1, 2, ..., $n-1$) are open. The N-th segment on the right is always closed.

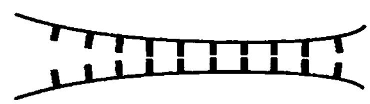


(a) [10 pts] Using a variable $\beta = \frac{1}{k_B T}$, show that the partition function for a canonical distribution has the form $Z_1(\beta) = \frac{1 - e^{-N\beta\epsilon}}{1 - e^{-\beta\epsilon}}$.

(b) [10 pts] Show that the mean energy of the system is written as $\langle E \rangle = -\frac{N\epsilon}{e^{N\beta\epsilon} - 1} + \frac{\epsilon}{e^{\beta\epsilon} - 1}$. Find the average number of open segments in the system, $\langle N_{open} \rangle$.

(c) [15 pts] Discuss the two limiting cases of $\langle E \rangle$ and $\langle N_{open} \rangle$ and their physical meanings when: (1) $\epsilon \gg k_B T$, and (2) $\epsilon \ll k_B T$.

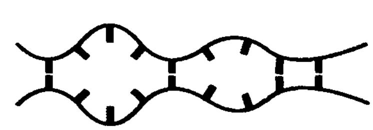
Now, consider the case where the zipper is allowed to unzip from both ends. Let us assume that at least one segment is always closed.



(d) [10 pts] Show that the partition function is now $Z_2(\beta) = \frac{1 - (N+1)e^{-N\beta\epsilon} + Ne^{-(N+1)\beta\epsilon}}{(1 - e^{-\beta\epsilon})^2}$. (Hint: One may use: $\frac{df(x)}{dx} = \sum_n n x^{n-1}$ if $f(x) = \sum_n x^n$.)

(e) [10 pts] Find the average number of open segments in this system in the $\epsilon \gg k_B T$ limit. Compare your result with (c). (Hint: One may use a relation $\ln(1+x) \approx x$ when $|x| \ll 1$ to simplify the calculation.)

Finally, consider the case where each segment could be open regardless of the states of its neighboring segments, or of any other segments in the zipper.

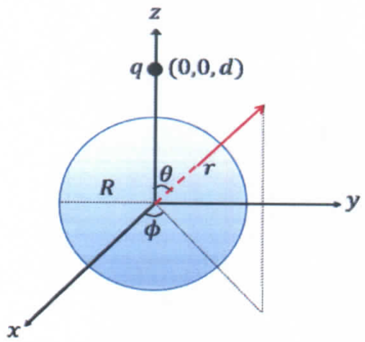


(f) [15 pts] Find the average number of open segments in this system. Compare your result with (c) for the limiting case of $\epsilon \gg k_B T$.

자격시험 문제

과목명 : 전기역학 2019 . 07. 19 시행

1. [30 pts] The main idea of the method of images is to add a set of fictitious charges so that together with the actual charges they satisfy the boundary condition of the original problem. Consider a point charge q at $(0,0,d)$ outside of a grounded conducting sphere of radius R at the center ($d > R$).



(a) [10 pts] Suppose we add a single image charge q' at a distance d' from the center. Find q' and its location.

(b) [5 pts] Find the potential at an arbitrary point outside of the sphere.

(c) [5 pts] Draw the electric field schematically. Assume $q > 0$.

(d) [5 pts] Find the surface charge density on the surface of the sphere at an arbitrary point.

(e) [5 pts] What is the total induced charge on the surface of the sphere?

2. [30 pts] Consider a particle of mass m and charge $-e$ ($e > 0$) located at the origin at time $t = 0$. For $t > 0$, an electric field E along the x direction and a magnetic field B along the z direction are applied.

(a) [10 pts] Write down the equation of motion for the particle. For the motion in the xy plane, introduce $R = x + iy$ and rewrite the equation of motion using R.

(b) [10 pts] Find the position of the particle at $t > 0$.

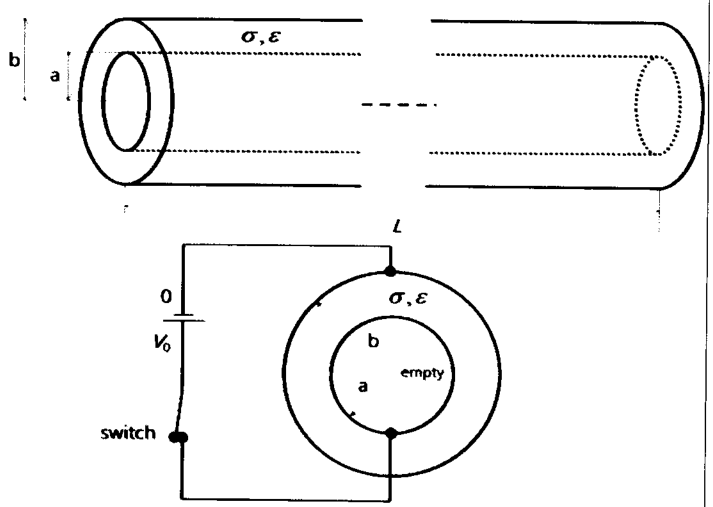
(c) [5 pts] The motion can be interpreted as a superposition of a cyclotron motion and a drift motion. Find the cyclotron frequency and the drift velocity.

(d) [5 pts] Draw the trajectory of the particle.

자격시험 문제

과목명 : 전기역학 2019 . 07. 19 시행

3. [40 pts] Two long highly conducting coaxial tubes (radii a and b with total length L and negligible thickness) are separated by a material of conductivity σ and dielectric constant ϵ. [The conductivity of the tube is much higher than σ, so each tube can be considered as an equipotential surface. Let us neglect fringing effects due to the edges and thickness of the tubes.] The tubes have potential difference V_0 maintained by a battery.



(a) [15 pts] Evaluate the capacitance C between the tubes.

(b) [15 pts] Evaluate the resistance R between the tubes What is the relationship between R and C ?

(c) [10 pts] If you disconnect the switch at t=0, the charge will gradually leak off. What is the potential difference V(t) across the tubes as a function of time ?

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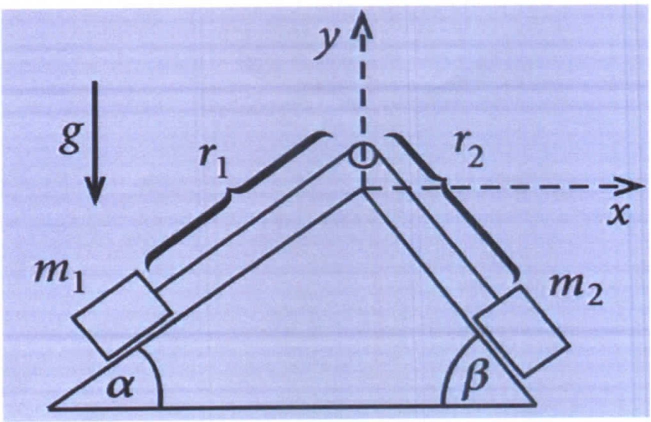
자격시험 문제

과목명 : 고전역학 2019 . 07. 19 시행

1. Two blocks of mass m_1 and m_2 move smoothly on a fixed wedge under the influence of gravity g. They are connected by a massless thread of length l.



(a) [5 pts] Write down the constraints in terms of x_1, y_1, x_2, and y_2 (For convenience, one might use r_1 and r_2). What is the number of degrees of freedom in this system?

(b) [5 pts] Choose generalized coordinate(s) from the result of (a) and express x_1, y_1, x_2, and y_2 in terms of the generalized coordinate(s) and l, α, β.

(c) [10 pts] Formulate the Lagrangian and obtain the Lagrange equation(s) of motion.

(d) [10 pts] Solve the Lagrange equation(s) for the following boundary conditions and find the equilibrium condition.

$$r_1(t=0) = r_0$$
$$\dot{r}_1(t=0) = 0$$

(e) [20 pts] Set up the Lagrangian in terms of r_1 and r_2. This time, introduce a Lagrangian multiplier λ to deal with constraints related to r_1 and r_2. Obtain Lagrange equations of motion and find the tension on the thread. What is the value of the tension using the equilibrium condition of (d)?

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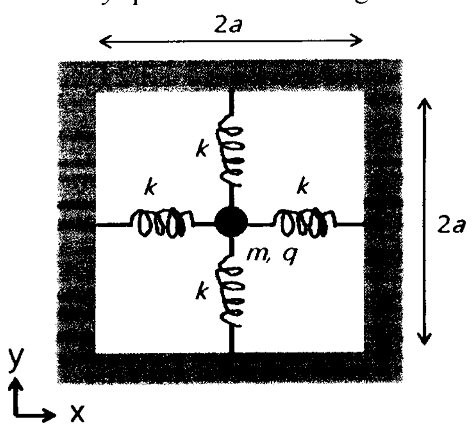
자격시험 문제

과목명 : 고전역학 2019 . 07. 19 시행

2. A particle of mass m and charge $q(>0)$ is placed in the xy plane as in the figure below.



The spring constant of each spring is k and its natural length $l < a$. Let's consider small oscillations of the particle around its equilibrium point.

(a) [8 pts] What are the oscillation frequency ω_0 of the particle?

(b) [10 pts] Now, a magnetic field B is applied along the $+z$ direction. Write down the equation of motion for the particle.

(c) [12 pts] Find the normal modes of small oscillations in (b) and their corresponding frequencies $\omega_{\pm}$, where $\omega_+ \geq \omega_-$. Show that the particle's motion under the magnetic field is generally expressed as

$$x(t) = A_- \cos(\omega_- t + \phi_-) + A_+ \cos(\omega_+ t + \phi_+).$$
$$y(t) = A_- \sin(\omega_- t + \phi_-) - A_+ \sin(\omega_+ t + \phi_+)$$

(d) [10 pts] Initially, the particle oscillates along the $x = y$ line as $x(t) = y(t) = A \sin(\omega_0 t)$ for zero magnetic field. At $t = 0$, the magnetic field B is suddenly turned on. Express the particle's motion after the field turn-on.

(e) [10 pts] Consider a situation where the magnetic field is very slowly ramped up to B instead of being suddenly turned on. Show that the subsequent motion of the particle satisfies the following relation,

$$\omega_+ |A_+| = \omega_- |A_-|.$$

Hint: You don't need to exactly calculate $A_{\pm}$.